9.4b Radius and Interval of Convergence

For what values of x is $\underbrace{\frac{1}{1+x^2}}_{f1} = \underbrace{1 - x^2 + x^4 - x^6}_{f2} + \ldots + (-1)^n x^{2n} + \ldots$ geo $a = 1$, $r = -x^2$

Do analytically and graphically.

$|r| < 1$

$|-x^2| < 1$

for convergence

$-1 < x < 1$ interval of convergence $(-1, 1)$

What are the radius and interval of convergence?

$|x^2| < 1$

$\sqrt{x^2} < \sqrt{1}$

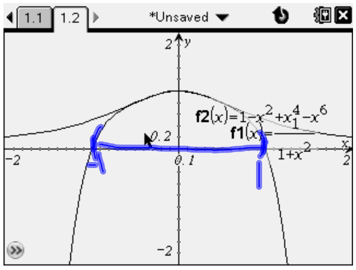
$|x| < 1$

$-1 < x < 1$

radius

$-1 \quad 0 \quad 1$

radius = 1



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Find the radius of convergence

$\sum_{n=0}^{\infty} \frac{nx^n}{10^n}$ not geometric, so use ratio test & abs convergence

$$\lim_{n\to\infty} \left| \frac{(n+1)\cancel{x^{n+1}}}{\cancel{10^{n+1}}} \cdot \frac{\cancel{10^n}}{n\cancel{x^n}} \right|$$

solve for x

$$\lim_{n\to\infty} \left| \frac{n+1}{n} \frac{x}{10} \right| = \frac{|x|}{10} < 1$$

1

$|x| < 10$ radius

$-10 < x < 10$

10

$-10 \quad 0 \quad 10$

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Find the radius of convergence

$$\sum_{n=0}^{\infty} n!x^n$$

ratio test

$$\lim_{n\to\infty} \left| \frac{(n+1)\cancel{!}\, x^{\cancel{n+1}}}{\cancel{n!}\, \cancel{x^n}} \right| = \infty$$

unless $x=0$

$-1 \quad 0 \quad 1$

radius $=0$

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Find the radius of convergence

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}x^n}{3^n}$$

ratio test

$$\lim_{n\to\infty} \left| \frac{\sqrt{n+1}\quad x^{\cancel{n+1}}}{3^{\cancel{n+1}}} \cdot \frac{\cancel{3^n}}{\sqrt{n}\ \cancel{x^n}} \right|$$

$$\lim_{n\to\infty} \left| \sqrt{\frac{n+1}{n}} \cdot \frac{x}{3} \right| = \frac{|x|}{3} < 1$$

1

$|x| < 3$

radius $=3$

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seq converges to 1
series diverges

34. $\sum_{n=1}^{\infty} n \sin \frac{1}{n} = 1 \sin 1 + 2 \sin \frac{1}{2} + 3 \sin \frac{1}{3} \cdots$

$\infty \cdot 0$

n^{th} term $\lim_{n\to\infty} n \sin \frac{1}{n}$

$\frac{0}{0}$ $\lim_{n\to\infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} =$

$\lim_{n\to\infty} \frac{\cos \frac{1}{n} \cancel{\left(-\frac{1}{n^2}\right)}}{\cancel{\left(-\frac{1}{n^2}\right)}} = 1$

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37. $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\, n!\, 3^n}$ converges

$(n+4)! = (n+4)(n+3)!$

$(n+1)! = (n+1) \cdot n!$

$\frac{3^n}{3^{n+1}} = \frac{1}{3}$

ratio test

$\lim_{n\to\infty} \frac{(n+4)\cancel{!}}{\cancel{3!}(n+1)\cancel{!}\, 3^{\cancel{n+1}}} \cdot \frac{\cancel{3!\, n!\, 3^n}}{\cancel{(n+3)!}}$

$\lim_{n\to\infty} \left(\frac{n+4}{n+1}\right) \frac{1}{3} = \frac{1}{3} < 1$

$\to 1$

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seq converges to e

38 $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n = 2+\left(\frac{3}{2}\right)^2+\left(\frac{4}{3}\right)^3 \ldots \to e$

series diverges

1^∞ $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$
$= y$

$\left(\frac{n+1}{n}\right)^n = \left(1+\frac{1}{n}\right)^n$

$\infty \cdot 0$ $\lim_{n\to\infty} n \ln\left(1+\frac{1}{n}\right) = \ln y$

$\frac{0}{0}$ $\lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{1}{1+\frac{1}{n}} \cdot \cancel{\left(\frac{-1}{n^2}\right)}}{\cancel{\left(\frac{-1}{n^2}\right)}} = 1$

$1 = \ln y$ $y = e$

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$\ln(a \cdot b) = \ln a + \ln b$

42 $\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$ converges

ratio test

$\lim_{n\to\infty} \frac{(n+1)\ln(n+1)}{2^{\cancel{n+1}}} \cdot \frac{\cancel{2^n}}{n \ln n}$

$\lim_{n\to\infty} \underbrace{\frac{n+1}{n}}_{\to 1} \cdot \underbrace{\frac{\ln(n+1)}{\ln n}}_{\to 1} \frac{1}{2} = \frac{1}{2} < 1$

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44. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges $(n+1)! = (n+1)n!$

ratio tests

$\lim_{n\to\infty} \frac{\cancel{(n+1)!}}{(n+1)^{n+1}} \cdot \frac{n^n}{\cancel{n!}}$

$\frac{n+1}{(n+1)^{n+1}}$

1^{∞} $\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \frac{1}{e} < 1$

$\frac{3^1}{3^{n+1}}$

$\frac{3^1}{3^5} = \frac{1}{3^4}$

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$\frac{1}{2} + \frac{1}{2} + \underbrace{\frac{1}{4}+\frac{1}{4}} + \underbrace{\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}}$

45 $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \cdots$

harmonic series

seq. converges to 0

series diverges

$\sum_{n=1}^{\infty} \frac{1}{n} > \sum_{n=1}^{\infty} \frac{1}{2}$

diverges diverges

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